



Mid Term Examination Second Semester 2015-2016 by Dr. Haythem H. Abullah

-Answer all the following questions

-Illustrate your answers with sketches when necessary

-No. of questions: 2

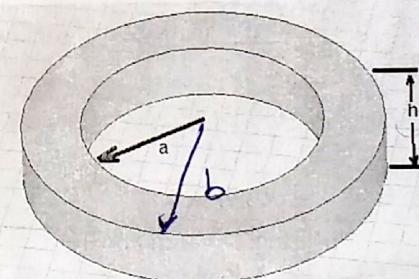
-Total marks: 20 marks

Question (1) (10 marks)

1. Given the points M(0.1,-0.2,-0.1), N(-0.2,0.1,0.3), and P(0.4,0,0.1), find a
 - a. The vector R_{MN}
 - b. The Dot product $R_{MN} \cdot R_{MP}$
 - c. The scalar projection of R_{MN} on R_{MP}
 - d. The angle between R_{MN} and R_{MP}(4 marks)
2. Find the vector component of $\vec{F} = 10\hat{a}_x - 6\hat{a}_y + 5\hat{a}_z$ that is parallel to $\vec{G} = 0.1\hat{a}_x + 0.2\hat{a}_y + 0.3\hat{a}_z$ and then find the vector component of \vec{F} that is perpendicular to \vec{G} (2 marks)
3. Four 10-nC positive charges are located in the z = 0 plane at the corners of a square 8 cm on a side. A fifth 10-nC positive charge is located at a point 8 cm distant from each of the other charges. Calculate the magnitude of the total force on this fifth charge for $\epsilon = \epsilon_0$ (4 marks)

Question (2) (10 marks)

1. Find the electric field due to an infinite sheet of charge carrying a surface charge density of ρ_s c/m² (4 marks)
2. Using Coulomb's law, find the electric field on a point on the z axis due to the volume of charge ρ_v c/m³ of the shell shown in the following figure. The z axis is perpendicular to shell. (3 marks)
3. Volume charge density is located in free space as $\rho_v = 2e^{-1000r} nC/m^3$ for $0 < r < 1 mm$, and $\rho_v = 0$ elsewhere.
 - a. Find the total charge enclosed by the spherical surface $r = 1 mm$
 - b. By using Gauss's law, calculate the value of D_r on the surface $r = 1 mm$ (3 marks)



Answer mid term second semester

2015-2016 for electromagnetic Basics

Q(1)

1

Given the Points $M(0.1, -0.2, -0.1)$

$N(-0.2, 0.1, 0.3)$

$P(0.4, 0, 0.1)$

Find ① the vector \vec{R}_{MN} .

$$\vec{R}_{MN} = (-0.2, 0.1, 0.3) - (0.1, -0.2, -0.1)$$

$$= -0.3 \hat{a}_x + 0.3 \hat{a}_y + 0.4 \hat{a}_z$$

② Dot Product $\vec{R}_{MN} \cdot \vec{R}_{MP}$

$$\vec{R}_{MP} = (0.4, 0, 0.1) - (0.1, -0.2, -0.1)$$

$$= 0.3 \hat{a}_x + 0.2 \hat{a}_y + 0.2 \hat{a}_z$$

$$\vec{R}_{MN} \cdot \vec{R}_{MP} = (-0.3, 0.3, 0.4) \cdot (0.3, 0.2, 0.2)$$
$$= 0.05$$

③ the scalar projection of $\vec{R}_{MN} \cdot \vec{R}_{MP}$

$$\vec{R}_{MN} \cdot \vec{a}_{\vec{R}_{MP}} = (-0.3, 0.3, 0.4) \cdot \frac{(0.3, 0.2, 0.2)}{\sqrt{0.09 + 0.04 + 0.04}}$$
$$\textcircled{1} = \frac{0.05}{\sqrt{0.17}} = \boxed{0.12}$$

the angle between \vec{R}_{MN} and \vec{R}_{MP}

$$\theta_m = \cos^{-1} \left(\frac{\vec{R}_{MN} \cdot \vec{R}_{MP}}{|\vec{R}_{MN}| |\vec{R}_{MP}|} \right)$$

$$= \cos^{-1} \left(\frac{0.05}{\sqrt{0.34} \sqrt{0.17}} \right) = 78^\circ$$

[2] find the vector component of $\vec{F} = 10\hat{a}_x - 6\hat{a}_y + 5\hat{a}_z$

that is parallel to $\vec{G} = 0.1\hat{a}_x + 0.2\hat{a}_y + 0.3\hat{a}_z$

and then find the vector component of \vec{F} that

is perpendicular to \vec{G}

$$\textcircled{1} \quad F_{||} = (\vec{F} \cdot \hat{G}) \hat{G}$$

$$\xrightarrow{\text{Parallel}} \vec{F} \cdot \vec{G} = (10\hat{a}_x - 6\hat{a}_y + 5\hat{a}_z) \cdot \frac{0.1\hat{a}_x + 0.2\hat{a}_y + 0.3\hat{a}_z}{\sqrt{(0.1)^2 + (0.2)^2 + (0.3)^2}}$$

$$= \frac{1 - 1.2 + 1.5}{\sqrt{0.14}} = \frac{1.3}{\sqrt{0.14}} = 3.47$$

$$F_{||} = (3.47) \cdot \hat{G} = 3.47 \cdot \frac{0.1\hat{a}_x + 0.2\hat{a}_y + 0.3\hat{a}_z}{\sqrt{0.14}}$$

$$= 3.47 \cdot (0.1\hat{a}_x + 0.2\hat{a}_y + 0.3\hat{a}_z)$$

(2)

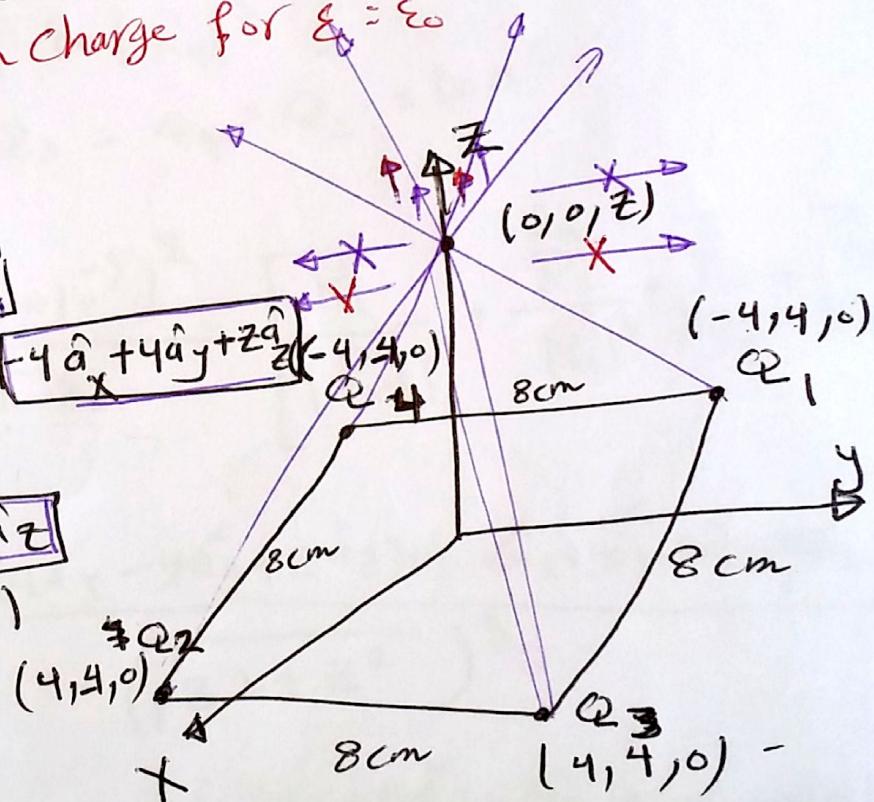
$$= [0.927\hat{a}_x + 1.854\hat{a}_y + 2.781\hat{a}_z]$$

$$\begin{aligned} \vec{F}_{\perp} &= \vec{F} - \vec{F}_{||} = (10\hat{a}_x - 6\hat{a}_y + 5\hat{a}_z) - (0.927\hat{a}_x + 1.85\hat{a}_y \\ &\quad + 2.781\hat{a}_z) \\ &\text{Perpendicular} \end{aligned}$$

$$= \boxed{9.073\hat{a}_x - 7.85\hat{a}_y + 2.219\hat{a}_z}$$

③ Four 10nc Positive charges are located in the $Z = 0$ Plane at the corners of a square 8 cm on side. A fifth 10nc positive charge is located at point $(0, 0, Z)$. Calculate the magnitude of the total force on this fifth charge for $\epsilon = \epsilon_0$

$$\begin{aligned} \vec{R}_1 &= (0, 0, Z) - (-4, 4, 0) \\ &= 4\hat{a}_x - 4\hat{a}_y + Z\hat{a}_z \\ \vec{R}_2 &= (0, 0, Z) - (4, 4, 0) = -4\hat{a}_x + 4\hat{a}_y + Z\hat{a}_z \\ \vec{R}_3 &= (0, 0, Z) - (4, -4, 0) \\ &= -4\hat{a}_x + 4\hat{a}_y + Z\hat{a}_z \\ \vec{R}_4 &= (0, 0, Z) - (-4, -4, 0) \\ &= 4\hat{a}_x + 4\hat{a}_y + Z\hat{a}_z \end{aligned}$$



$$|\vec{R}_1| = \sqrt{32 + z^2} = \sqrt{16 + 16 + z^2}$$

$$|\vec{R}_2| = \sqrt{32 + z^2}$$

$$|\vec{R}_3| = \sqrt{32 + z^2}$$

$$|\vec{R}_4| = \sqrt{32 + z^2}$$

$$\vec{F}_{\text{total}} = \vec{F}_{15} + \vec{F}_{25} + \vec{F}_{35} + \vec{F}_{45}$$

$$= \frac{Q_1 Q_5 \vec{R}_1}{4\pi \epsilon_0 |\vec{R}_1|^3} + \frac{Q_2 Q_5 \vec{R}_2}{4\pi \epsilon_0 |\vec{R}_2|^3} + \frac{Q_3 Q_5 \vec{R}_3}{4\pi \epsilon_0 |\vec{R}_3|^3} + \frac{Q_4 Q_5 \vec{R}_4}{4\pi \epsilon_0 |\vec{R}_4|^3}$$

$$\therefore Q_1 = Q_2 = Q_3 = Q_4 = Q_5 = 10 \text{ nC}$$

$$\therefore F_{\text{total}} = \frac{(10 * 10^{-9})^2}{4\pi \epsilon_0} \left[\frac{\vec{R}_1}{|\vec{R}_1|^3} + \frac{\vec{R}_2}{|\vec{R}_2|^3} + \frac{\vec{R}_3}{|\vec{R}_3|^3} + \frac{\vec{R}_4}{|\vec{R}_4|^3} \right]$$

$$\therefore F_{\text{total}} = \frac{(10 * 10^{-9})^2}{4\pi \epsilon_0} \left[\frac{(4\hat{a}_x - 4\hat{a}_y + z\hat{a}_z) + (-4\hat{a}_x + 4\hat{a}_y + z\hat{a}_z)}{(\sqrt{32 + z^2})^3} \right. \\ \left. + (-4\hat{a}_x + 4\hat{a}_y + z\hat{a}_z) + (4\hat{a}_x + 4\hat{a}_y + z\hat{a}_z) \right]$$

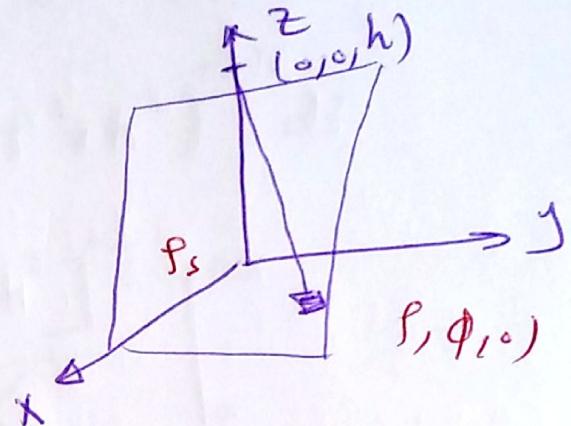
$$F_{\text{total}} = \frac{(10 * 10^{-9})}{4\pi \epsilon_0} \left[\frac{8\hat{a}_y + 4z\hat{a}_z}{\sqrt{32 + z^2}} \right] \checkmark \quad \checkmark$$

(4)

Q₂

- ① Find the electric field due to an infinite sheet of charge carrying a surface charge density of $\rho_s \text{ C/m}^2$

$$\vec{E} = \frac{\iint \rho_s dS \hat{R}}{4\pi \epsilon_0 R^2}$$



$$\begin{aligned}\vec{R} &= \vec{r}_P - \vec{r}_s = (0, 0, h) - (\rho, \phi, 0) \\ &= -\rho \hat{a}_\rho + h \hat{a}_z\end{aligned}$$

$$|\vec{R}| = \sqrt{\rho^2 + h^2}$$

$$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{-\rho \hat{a}_\rho + h \hat{a}_z}{\sqrt{\rho^2 + h^2}}$$

$$dS \perp z$$

$$dS = \rho d\rho d\phi$$

$$\vec{E} = \int_0^\infty \int_0^{2\pi} \frac{\rho_s \rho d\phi d\rho [-\rho \hat{a}_\rho + h \hat{a}_z]}{4\pi \epsilon_0 [\rho^2 + h^2]^{3/2}}$$

$$= \frac{\rho_s 2\pi}{4\pi \epsilon_0} \int_0^\infty \frac{\rho d\rho [-\rho \hat{a}_\rho + h \hat{a}_z]}{[\rho^2 + h^2]^{3/2}}$$

صفر على \hat{a}_ρ لذا $\int_0^\infty \rho d\rho = 0$

$$\begin{aligned}
 \vec{E} &= \frac{\rho_0 s}{2\epsilon_0} \int_0^\infty \frac{h \hat{a}_z ds}{[s^2 + h^2]^{3/2}} \\
 &= \frac{\rho_0 s}{2\epsilon_0} \hat{a}_z \int_0^\infty (s^2 + h^2)^{-3/2} ds \\
 &= \frac{h \rho_0 s}{2\epsilon_0} \hat{a}_z * \frac{1}{2} \int_0^\infty 2s (s^2 + h^2)^{-3/2} ds \\
 &= \frac{h \rho_0 s}{2\epsilon_0} \hat{a}_z * \frac{1}{2} \left[\frac{s^2 + h^2}{-1/2} \right] \Big|_0^\infty
 \end{aligned}$$

$$= \frac{k \rho_0 s}{2\epsilon_0} \hat{a}_z \left[\frac{1}{\sqrt{s^2 + h^2}} \Big|_0^\infty + \frac{1}{\sqrt{k^2 + h^2}} \right]$$

$\vec{E} = \frac{\rho_0 s}{2\epsilon_0} (\pm \hat{a}_n)$

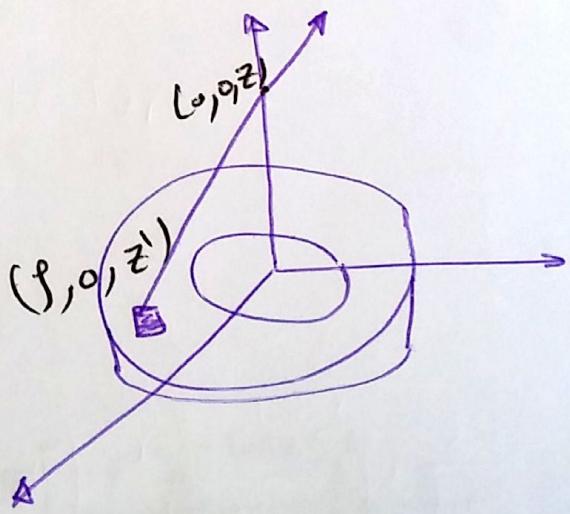
* ----- *

(2) using coulomb's law find the electric field on a point on the z-axis due to the volume of charge $\rho_v \text{ C/m}^3$

$$\vec{E} = \iiint \frac{\rho_v s d\rho d\phi dz}{4\pi \epsilon_0 R^3} \vec{R}$$

$$\vec{R} = -s \hat{a}_\rho + (z - z') \hat{a}_z$$

$$R = \sqrt{s^2 + (z - z')^2}$$



(6)

$$\vec{E} = \int_0^h \int_0^{2\pi} \int_a^b \frac{\rho_v \rho d\rho d\phi dz (z-z') \hat{az}}{4\pi \epsilon_0 ((\rho^2 + (z-z')^2)^{3/2})}$$

Q3) Volume charge density is located in free space as $\rho_v = 2e^{-1000r} \text{ A/cm}^3$ for $0 < r < 1 \text{ mm}$ $\rho_v = 0$ elsewhere
find the total charge enclosed by the spherical surface $r = 1 \text{ mm}$.

$$\rho_v = 2e^{-1000r}$$

$$Q = \iiint_V \rho_v dV = \int_0^{2\pi} \int_0^\pi \int_0^1 2e^{-1000r} r^2 \sin\theta d\theta d\phi dr$$

$$Q = 8\pi \left[-\frac{r^2 e^{-1000r}}{1000} \Big|_0^1 + \frac{2}{1000} \cdot \frac{e^{-1000r}}{(1000)^2} (-1000r - 1) \right]$$

b) By using Gauss's law calculate D_r .

$$\Phi = \oint \vec{D} \cdot d\vec{s} = Q_{\text{enc}}$$

$$\vec{D}_r \cdot 4\pi r^2 = \iiint_V \rho_v dV = 8\pi \left[-\frac{r^2 e^{-1000r}}{1000} \Big|_0^1 + \frac{2}{1000} \cdot \frac{e^{-1000r}}{(1000)^2} (-1000r - 1) \Big|_0^1 \right]$$